CDA 3200 Digital Systems

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Outline

- Basic Operations of Boolean Algebra
- Examples
- Basic Theorems
- Commutative, Associative, and Distributive Laws
- Simplification Theorems
- Multiplying Out and Factoring
- DeMorgan’s Laws

Basic Operations of Boolean Algebra (1/11)

- All the switching devices are two-state devices.
- Boolean algebra is useful in analyzing switching devices and circuits.

Basic Operations of Boolean Algebra (2/11)

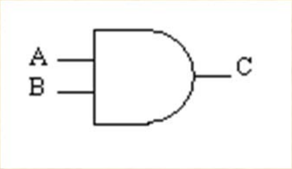
- Basic elements:
  - Three tools in analyzing switching devices
    - Boolean expression: $A+B$, $A \cdot B$
    - Truth table
    - Logic diagram
  - Inputs and outputs can only be 0's or 1's.

Basic Operations of Boolean Algebra (3/11)

- Basic operations:
  - AND
    - Logic expression: C=A·B or C=AB
    - Truth table

A	B	C=A·B
0	0	0
0	1	0
1	0	0
1	1	1

    - Logic diagram

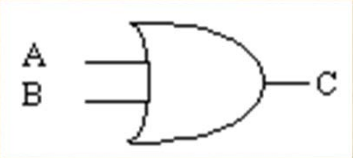


Basic Operations of Boolean Algebra (4/11)

- Basic operations (cont)
  - OR
    - Logic Expression: C=A+B
    - Truth Table

A	B	C=A·B
0	0	0
0	1	1
1	0	1
1	1	1

    - Logic diagram

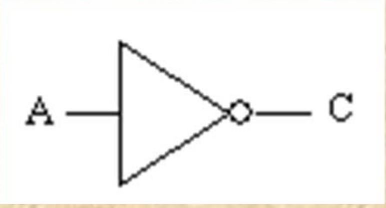


Basic Operations of Boolean Algebra (5/11)

- Basic operations (cont)
  - complement
    - Logic Expression: C=A’
    - Truth Table

A	C
0	1
1	0

    - Logic diagram

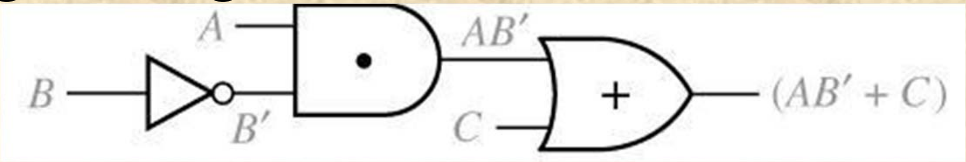


Basic Operations of Boolean Algebra (6/11)

- Rules of precedence:
  - Brackets
  - NOT
  - AND
  - OR

Basic Operations of Boolean Algebra (7/11)

- Example 1:
  - Logic expression
    - AB’+C
    - Order of execution: B’→AB’→AB’+C
  - Logic diagram



Basic Operations of Boolean Algebra (8/11)

- Example 1 (cont)
  - Truth Table
    - How many inputs: three
    - How many rows: $2^3=8$
    - How many outputs: one

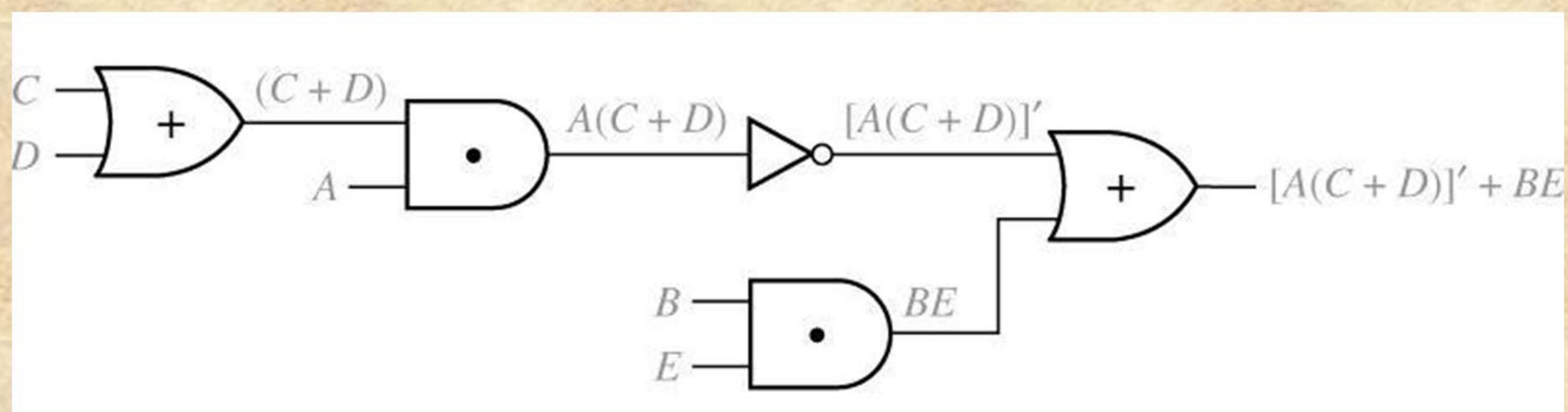
Basic Operations of Boolean Algebra (9/11)

- Example 1 (cont)

A B C	B’	AB’	AB’+C
0 0 0	1	0	0
0 0 1	1	0	1
0 1 0	0	0	0
0 1 1	0	0	1
1 0 0	1	1	1
1 0 1	1	1	1
1 1 0	0	0	0
1 1 1	0	0	1

Basic Operations of Boolean Algebra (10/11)

- Example 2:
  - Boolean expression: [A(C+D)]'+BE
  - Logic diagram:



Basic Operations of Boolean Algebra (11/11)

- Example 2
  - Truth Table
    - How many inputs: five
    - How many rows: 2^5=32
    - How many outputs: one

Basic Theorems

- Operations with 0 and 1
  - $X+0=X$ $X\cdot 1=X$
  - $X+1=1$ $X\cdot 0=0$
- Idempotent laws
  - $X+X=X$ $X\cdot X=X$
- Involution law
  - $(X')'=X$
- Laws of complementarity
  - $X+X'=1$ $X\cdot X'=0$

Commutative, Associative, and Distributive Laws

- Commutative laws
  - XY=YX
  - X+Y=Y+X
- Associative laws
  - (XY)Z=X(YZ)
  - (X+Y)+Z=X+(Y+Z)=X+Y+Z
  - XYZ=1 iff X=Y=Z=1
  - X+Y+Z=0 iff X=Y=Z=0
- Distributive laws
  - X(Y+Z)=XY+XZ
  - <u>X+YZ=(X+Y)(X+Z)</u>

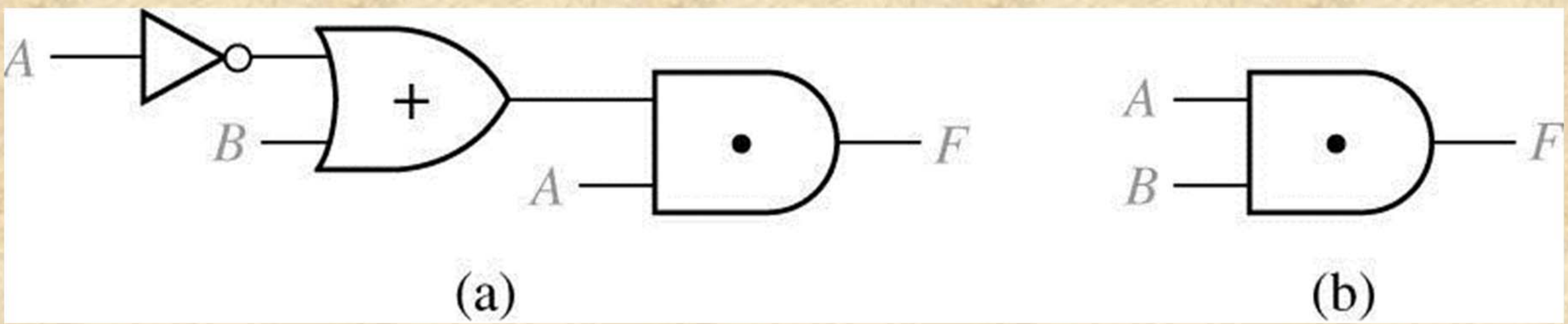
Simplification Theorems (1/3)

- $XY+XY'=X$
- $(X+Y)(X+Y')=X$

- $X+XY=X$
- $X(X+Y)=X$

- $(X+Y')Y=XY$
- $XY'+Y=X+Y$

Simplification Theorems (2/3)



Simplification Theorems (3/3)

- Z=[A+B’C+D+EF][A+B’C+(D+EF)’]
  - X=A+B’C and Y=D+EF
  - Z=(X+Y)(X+Y’)=X=A+B’C

- Z=(AB+C)(B’D+C’E’)+(AB+C)’
  - X=B’D+C’E’ and Y=(AB+C)’
  - Z=Y’X+Y=Y+X=B’D+C’E’+(AB+C)’

Multiplying and Factoring (1/4)

- Sum-of-products
  - All products are the products of single variables or complements.
  - AB’+CD’E+AC’E’
  - A+B’+C+D’E
  - (A+B)CD+EF X
- Product-of-sums
  - All sums are the sums of single variables or complements.
  - (A+B’)(C+D’+E)(A+C’+E’)
  - (A+B)(C+D+E)F
  - (A+B)(B’C+D) X

Multiplying and Factoring (2/4)

- Multiplying
  - (A+B)(B+C)(D’+B)(ACD’+E)
  - =(ACD’+B)(ACD’+E)
  - =ACD’+BE

Multiplying and Factoring (3/4)

- Factoring
  - AB’+C’D
  - =(AB’+C’)(AB’+D)
  - =(A+C’)(B’+C’)(A+D)(B’+D)

Multiplying and Factoring (4/4)

- A sum-of-products expression can always be realized directly by one or more AND gates feeding a single OR gate at the circuit output.

- A product-of-sums expression can always be realized by one or more OR gates feeding a single AND gate at the circuit output.

DeMorgan's Laws (1/2)

- $(X+Y)'=X'Y'$
- $(XY)'=X'+Y'$

- $(X_1+ X_2+ X_3+\ldots+ X_n)'=X_1' X_2' X_3'\ldots X_n'$
- $(X_1 X_2 X_3\ldots X_n)'= X_1'+ X_2'+ X_3'+\ldots+ X_n'$

DeMorgan's Laws (2/2)

- [(A'+B)C']'=(A'+B)'+(C')'=AB'+C

- [(A+B')C']'(A+B)(C+A)'
  - =(A'B+C)(A+B)A'C'
  - =(A'B+C)(A'BC')
  - =A'BC'