CDA 3200 Digital Systems

Instructor: Dr. Janusz Zalewski

Developed by: Dr. Dahai Guo

Spring 2012

Outline

- Multiplying Out and Factoring Expressions
- Exclusive-OR and Equivalence Operations
- The Consensus Theorem
- Algebraic Simplification of Switching Expressions
- Proving Validity of an Equation

Multiplying Out and Factoring Expressions (1/5)

- Theorem 3-1 X(Y+Z)=XY+XZ
- Theorem 3-2 (X+Y)(X+Z)=X+YZ

- Theorem 3-3 (X+Y)(X'+Z)=XZ+X'Y

Multiplying Out and Factoring Expressions (2/5)

- How can you prove (X+Y)(X'+Z)=XZ+X'Y is true?
  - Truth table
  - (X+Y)(X'+Z)
  - =XX'+XZ+X'Y+YZ   // XX'=0
  - =0+XZ+X'Y+YZ=XZ+X'Y+YZ
  - =XZ+X'Y+YZ(X+X')
  - =XZ+X'Y+XZY+X'YZ   // X+XY=X
  - =XZ+X'Y

Multiplying Out and Factoring Expressions (3/5)

- Theorem 3-3 (X+Y)(X'+Z)=XZ+X'Y can be useful for multiplying out expressions.
  - It is important to look for two terms, one of which contains a variable and another contains its complement.
  - (Q+AB')(C'D+Q')=QC'D+Q'AB'

Multiplying Out and Factoring Expressions (4/5)

- Generally, (X+Y)(X+Z)=X+YZ (3-2) and (X+Y)(X'+Z)=XZ+X'Y (3-3) are applied before X(Y+Z)=XY+XZ (3-1) in multiplying out expressions
  - (A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C) (Use 3-2)
  - =(A+B+C'DE)(A+D'+E)(A'+C) (Use 3-3)
  - =(A+B+C'DE)(AC+A'D'+A'E) (Use 3-1)
  - =AC+ABC+A'BD'+A'BE+A'C'DE (Use X+XY=X)
  - =AC+A'BD'+A'BE+A'C'DE

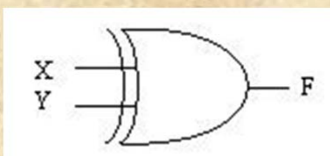
Multiplying Out and Factoring Expressions (5/5)

- If Theorem 3-1 was applied first, 162 would have been created!!
  - (A+B+C’)(A+B+D)(A+B+E)(A+D’+E)(A’+C)
  - 3 x 3 x 3 x 3 x 2 terms
  - =162 terms

Exclusive-OR and Equivalence Operations (1/4)

- Exclusive-OR
  - A binary operator

$X \oplus Y$



X	Y	F
0	0	0
0	1	1
1	0	1
1	1	0

When the two inputs are different, the output is one; otherwise it is zero.

Exclusive-OR and Equivalence Operations (2/4)

- Using AND and OR to implement exclusive-OR
  - $(X+Y)(XY)'=(X+Y)(X'+Y')$
  - $=XY'+X'Y$

Exclusive-OR and Equivalence Operations (3/4)

- The following theorems apply to exclusive-OR:

$$X \oplus 0 = X$$

$$X \oplus 1 = X$$

$$X \oplus X = 0$$

$$X \oplus X' = 1$$

$$X \oplus Y = Y \oplus X$$

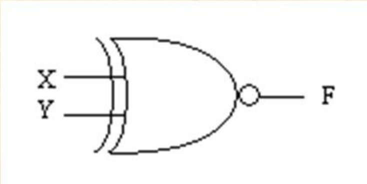
$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$$

$$X(Y \oplus Z) = XY \oplus XZ$$

$$(X \oplus Y)' = X \equiv Y = XY + X'Y'$$

Exclusive-OR and Equivalence Operations (4/4)

- The complement of exclusive-OR is the equivalence operation, also called exclusive-NOR
  - X≡Y



X	Y	F
0	0	1
0	1	0
1	0	0
1	1	1

The Consensus Theorem (1/6)

- Theorem 3-3: (X'+Y)(X+Z)=XY+X'Z
- In the derivation, we had
  - (X+Y)(X'+Z)
  - =XX'+XZ+X'Y+YZ
  - =0+XZ+X'Y+YZ=XZ+X'Y+YZ
  - =XZ+X'Y+YZ(X+X')
  - =XZ+X'Y+XZY+X'YZ
  - =XZ+X'Y
- Then we can have XZ+X'Y+YZ=XZ+X'Y
- The eliminated term, YZ, is referred to as the ***consensus term***.

The Consensus Theorem (2/6)

- How to recognize a consensus term
  - Find a pair of terms: one contains a variable and the other contains its complement
  - The consensus term is the addition of the two terms leaving out the selected variable and its complement.

The Consensus Theorem (3/6)

- A'C'D+A'BD+BCD+ABC+ACD'
- =A'C'D+A'BD+ABC+ACD' (BCD is eliminated)
- Or
- A'C'D+A'BD+BCD+ABC+ACD'
- =A'C'D+BCD+ABC+ACD' (A'BD is eliminated)
- =A'C'D+BCD+ACD' (ABC is eliminated)

The Consensus Theorem (4/6)

- Adding a term using the consensus theorem
  - F=ABCD+B'CDE+A'B'+BCE'
  - =ABCD+B'CDE+A'B'+BCE'+ACDE
  - =B'CDE+A'B'+BCE'+ACDE (ABCD is eliminated)
  - =A'B'+BCE'+ACDE (B'CDE is eliminated)

The Consensus Theorem (5/6)

- It is not a systematic method to simply logic expressions by finding consensus terms.

The Consensus Theorem (6/6)

- The dual form of the consensus theorem is
  - $(X+Y)(X'+Z)(Y+Z)=(X+Y)(X'+Z)$

Algebraic Simplification of Switching Expressions (1/7)

- Three basic ways of simplifying switching functions:
  - Combining terms:
    - $XY+XY'=X$
  - Eliminating terms:
    - $X+XY=X$
    - the consensus theorem
  - Eliminating literals:
    - $X+X'Y=X+Y$

Algebraic Simplification of Switching Expressions (2/7)

- Combining terms using XY+XY'=X
  - The two terms to be combined should contain exactly the same variables.
  - Exactly one of the variables should appear complemented in one term and not in the other.

Algebraic Simplification of Switching Expressions (3/7)

- Combining terms using XY+XY'=X (cont)
  - abc'd'+abcd'=abd'
  - ab'c+abc+a'bc= ab'c+abc+abc+a'bc=ac+bc
  - Sometimes, X and Y are replaced with more complicated expressions
    - (a+bc)(d+e')+a'(b'+c')(d+e')=d+e'

Algebraic Simplification of Switching Expressions (4/7)

- Eliminating terms:
  - To find a common part in the products: X+XY=X
    - a’b+a’bc=a’b
    - a+ab’c+abc’+ab’c’=a
  - To find consensus terms
    - a’bc’+bcd+a’bd=a’bc’+bcd
    - abce+b’cd+a’cd+cde
    - =abce+(a’+b’)cd+cde
    - =abce+(ab)’cd+cde
    - =abce+(ab)’cd (cde is eliminated)

Algebraic Simplification of Switching Expressions (5/7)

- Eliminating literals: Y+XY'=X+Y
  - A'B+A'B'C'D'+ABCD'
  - =A'(B+B'C'D')+ABCD' (Y+XY'=X+Y)
  - =A'(B+C'D')+ABCD'
  - =A'B+A'C'D'+ABCD'
  - =B(A'+ACD')+A'C'D' (Y+XY'=X+Y)
  - =B(A'+CD')+A'C'D'
  - =A'B+BCD'+A'C'D'

Algebraic Simplification of Switching Expressions (6/7)

- Eliminating literals: Y+XY’=X+Y (cont)
  - a’bd+abcd
  - =bd(a’+ac)
  - =bd(a’+c)
  - =a’bd+bcd

Algebraic Simplification of Switching Expressions (7/7)

- Adding redundant terms
  - WX+XY+X'Z'+WY'Z'
  - =WX+XY+X'Z'+WY'Z'+WZ'
  - =WX+XY+X'Z'+WZ' (WZ' is still a consensus term)
  - =WX+XY+X'Z'

Limitations

- The simplifying methods introduced in this chapters do not guarantee simplest logic expressions.
- These methods are not systematic.
- But they are still useful to do some simple logic analysis.