CDA 3200 Digital Systems

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Outline

- Minimum Forms of Switching Functions
- Two- and Three-Variable Karnaugh Maps
- Four-Variable Karnaugh Maps

Minimum Forms of Switching Functions (1/5)

- A *minimum sum-of-products* expression for a function is designed as a sum of product terms which
  - Has a minimum number of terms
  - Has a minimum number of literals

Minimum Forms of Switching Functions (2/5)

- The logic algebraic techniques can be used to simplify a logic expression to its minimum sum-of-products.
- However, the procedures are difficult to apply in a systematic way and it is difficult to tell when you have arrived at a minimum solution.

Minimum Forms of Switching Functions (3/5)

- Given a minterm expansion, the minimum sum-of-products form can often be obtained by the following procedure:
  - Combine terms by using XY'+XY=X to eliminate as many terms as possible.
  - Eliminate redundant terms by using the consensus theorem or other theorems.
- The result may depend on the order in which terms are combined or eliminated.

Minimum Forms of Switching Functions (4/5)

- Example:
  - F(a,b,c)=sum[m(0,1,2,5,6,7)]
  - =a’b’c’+a’b’c+a’bc’+ab’c+abc’+abc
  - =a’b’c’+a’b’c+a’b’c+a’bc’+ab’c+abc’+abc’+abc
  - =a’b’+b’c+bc’+ab

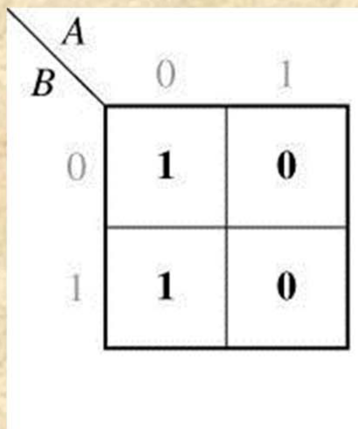
Minimum Forms of Switching Functions (5/5)

- Example: (cont)
  - F(a,b,c)=sum[m(0,1,2,5,6,7)]
  - =a'b'c'+a'b'c+a'bc'+ab'c+abc'+abc
  - =a'b'+bc'+ac

Two- and Three-Variable Karnaugh Maps (1/10)

- In a Karnaugh map, minterms in adjacent squares of the map can be combined since they differ in only one variable. The combinable terms are looped in the Karnaugh map.

A B	F
0 0	1
0 1	1
1 0	0
1 1	0



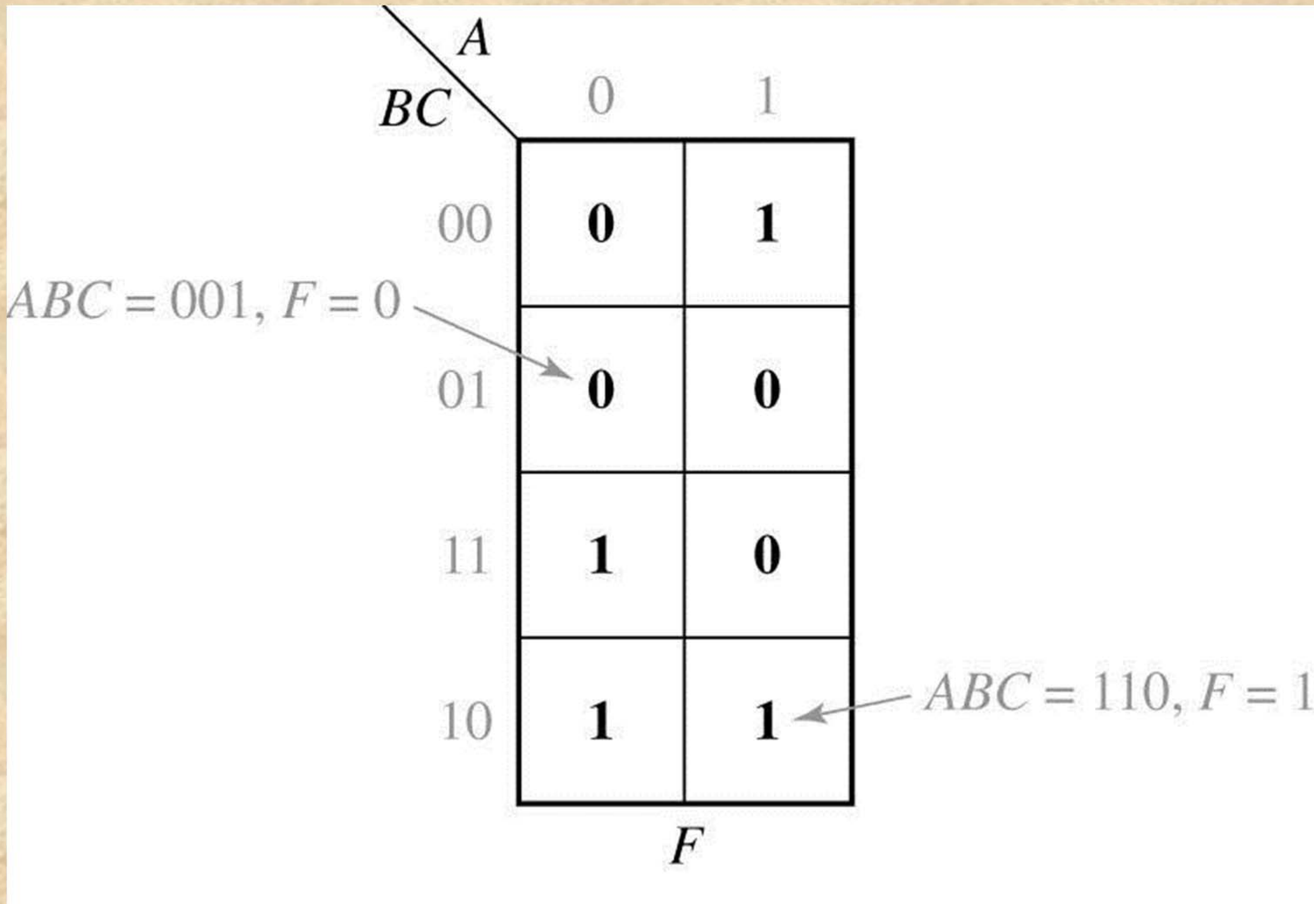
(b)

Two- and Three-Variable Karnaugh Maps (2/10)

- In a three-variable (A,B,C) Karnaugh map, the value of one variable A is listed across the top of the map, and the values of the other two variables (B,C) are listed along the side of the map.
- Note the rows are labeled in the sequence 00, 01, 11, 10, why?

Two- and Three-Variable Karnaugh Maps (3/10)

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

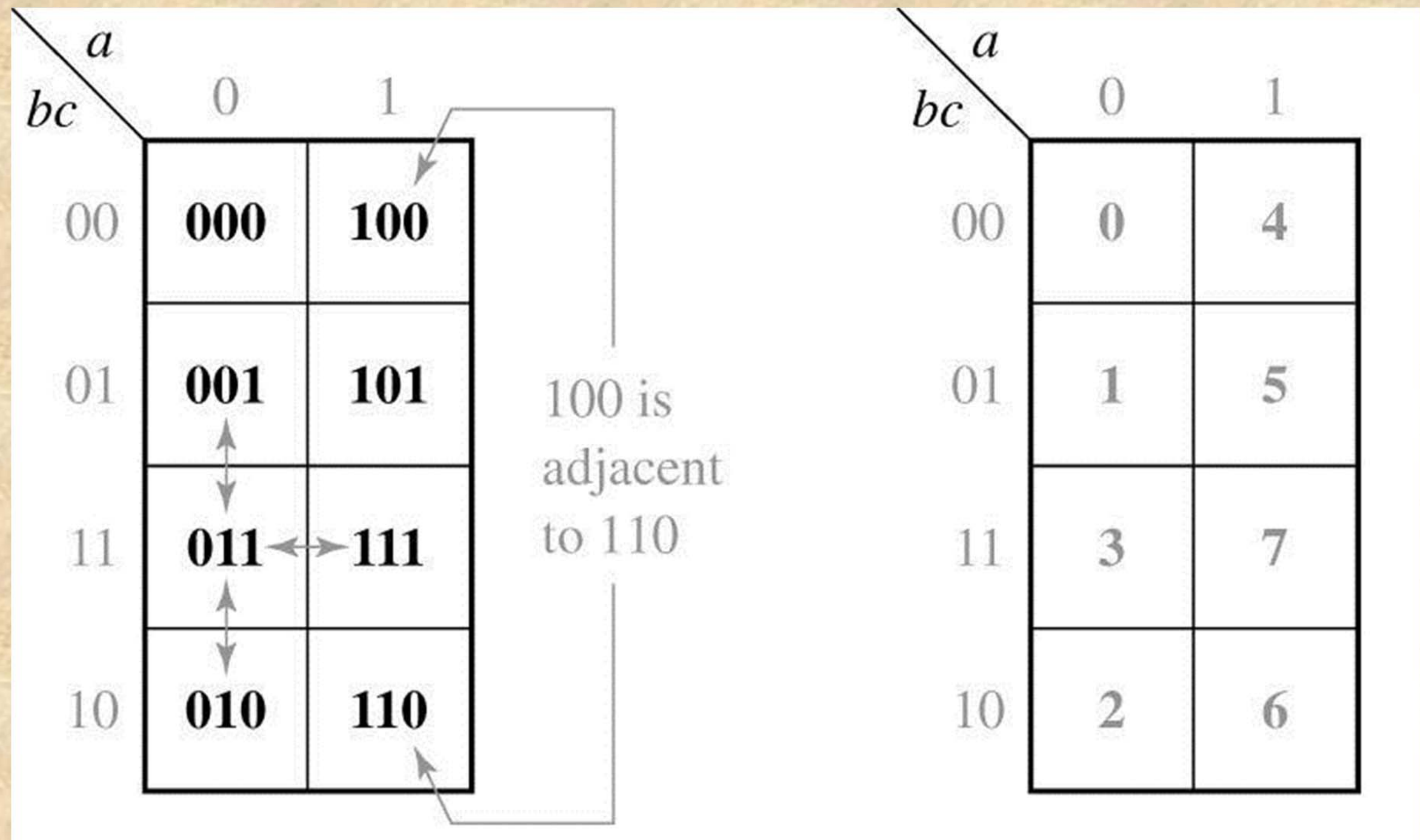


(b)

Two- and Three-Variable Karnaugh Maps (4/10)

- In a three-variable Karnaugh map, the top and bottom rows of the map are defined to be adjacent because the corresponding minterms in these rows differ in only one variable.

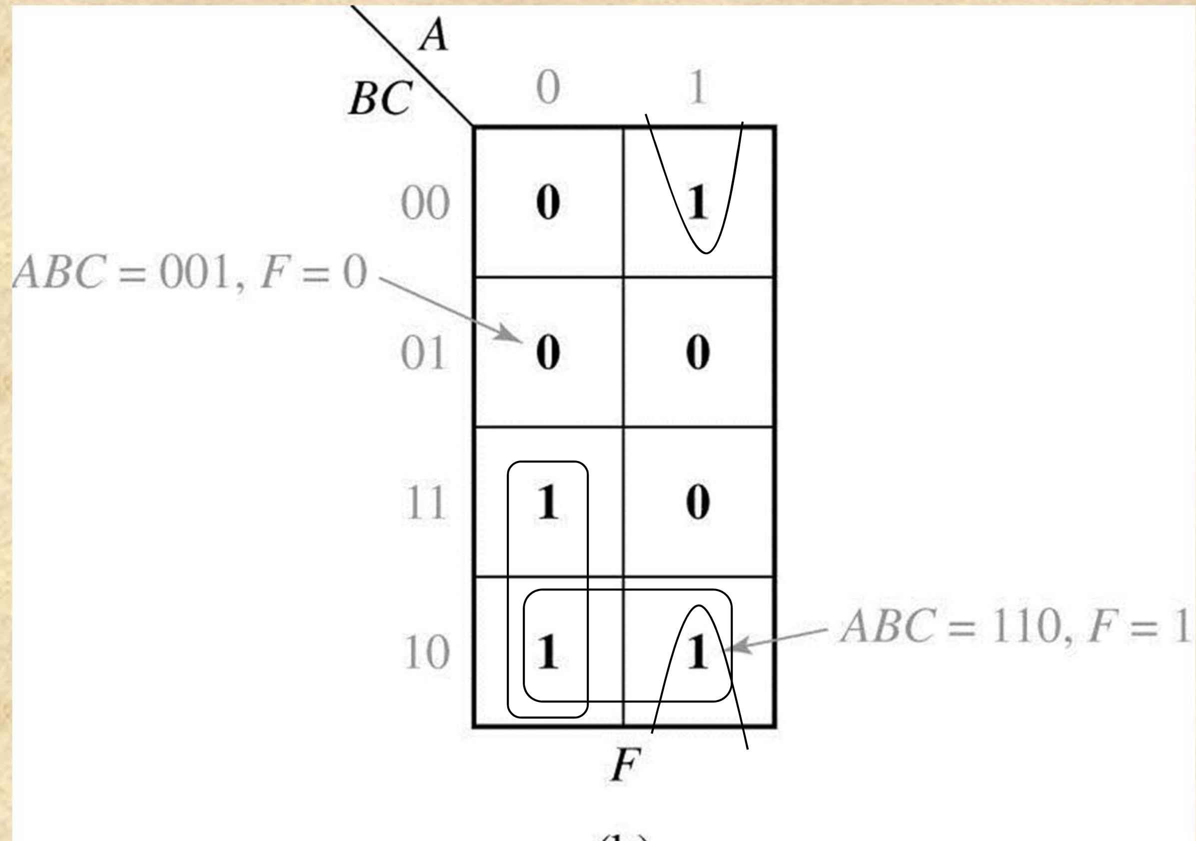
Two- and Three-Variable Karnaugh Maps (5/10)



(a) Binary notation (b) Decimal notation

Two- and Three-Variable Karnaugh Maps (6/10)

- How would you loop minterms in this Karnaugh map?



(b)

Two- and Three-Variable Karnaugh Maps (7/10)

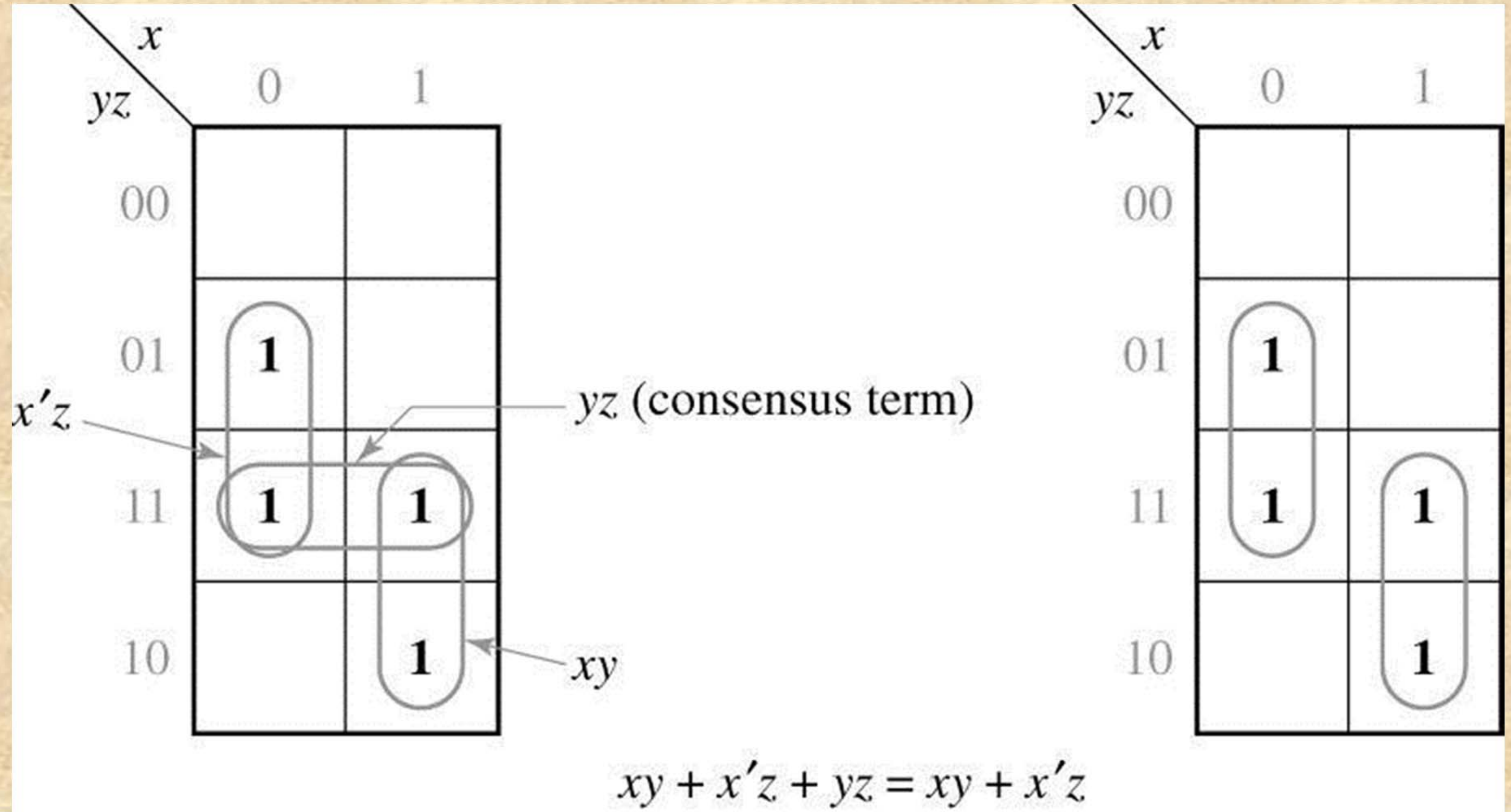
- How to plot 1s in a Karnaugh map for the following expressions and loop all you can loop:
  - F(a,b,c)=a'bc+abc'+abc+a'bc'
  - F(a,b,c)=abc'+b'c+a'
  - F(a,b,c)=b'c'+ab+bc'
  - F(a,b,c)=ab+a'c

Two- and Three-Variable Karnaugh Maps (8/10)

- <u>Two</u> terms in adjacent squares on the map differ in only one variable and can be combined using the theorem XY’+XY=X
- <u>Two</u> adjacent “loops” that differ only one variable can be combined.

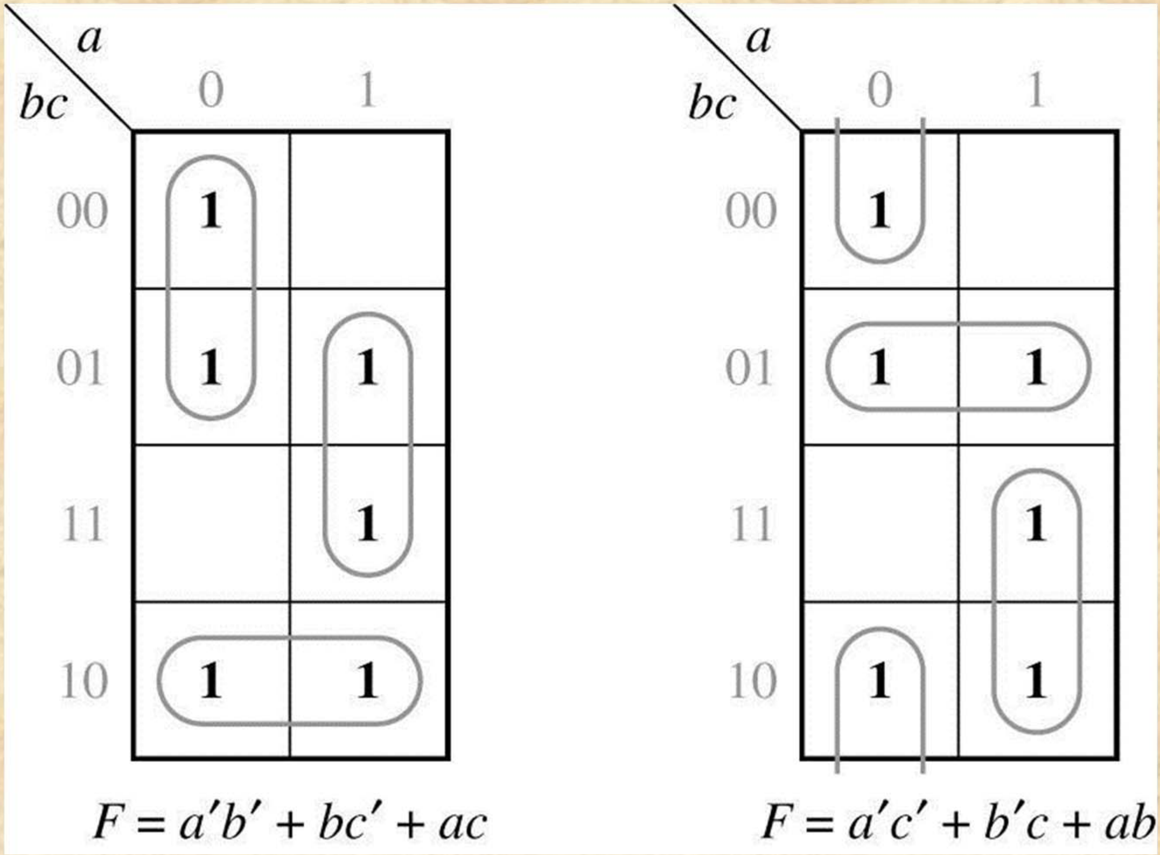
Two- and Three-Variable Karnaugh Maps (9/10)

- The Karnaugh map can also illustrate the consensus theorem XY+X'Z+YZ=XY+X'Z



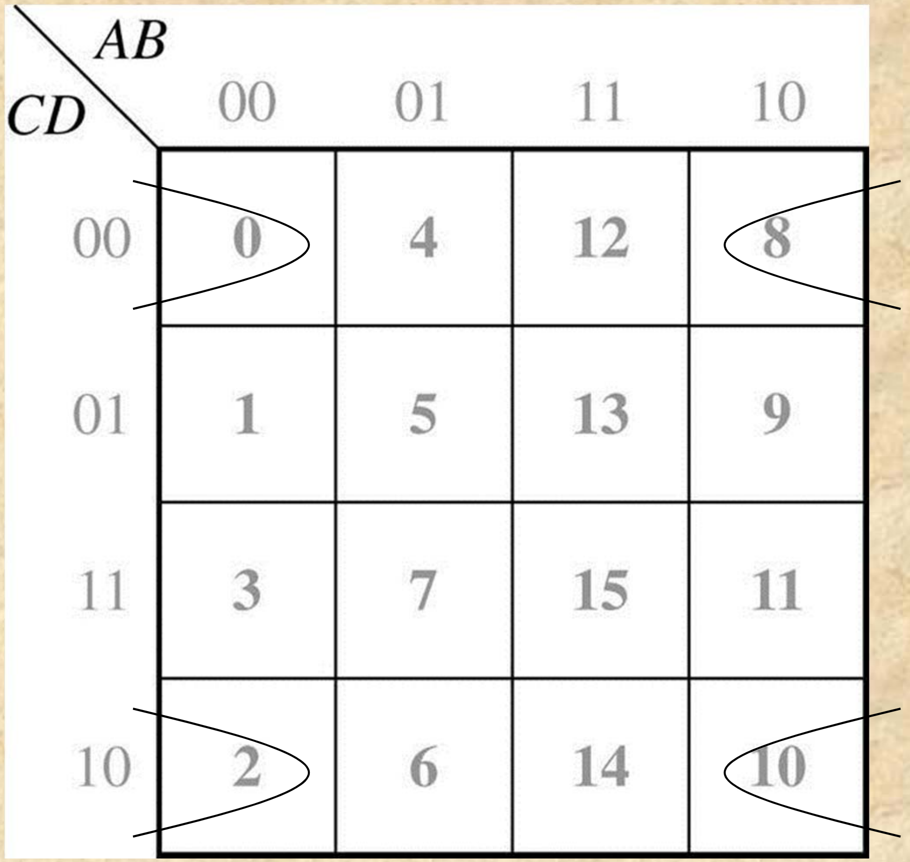
Two- and Three-Variable Karnaugh Maps (10/10)

- The simplification using Karnaugh maps can also result in different solutions.



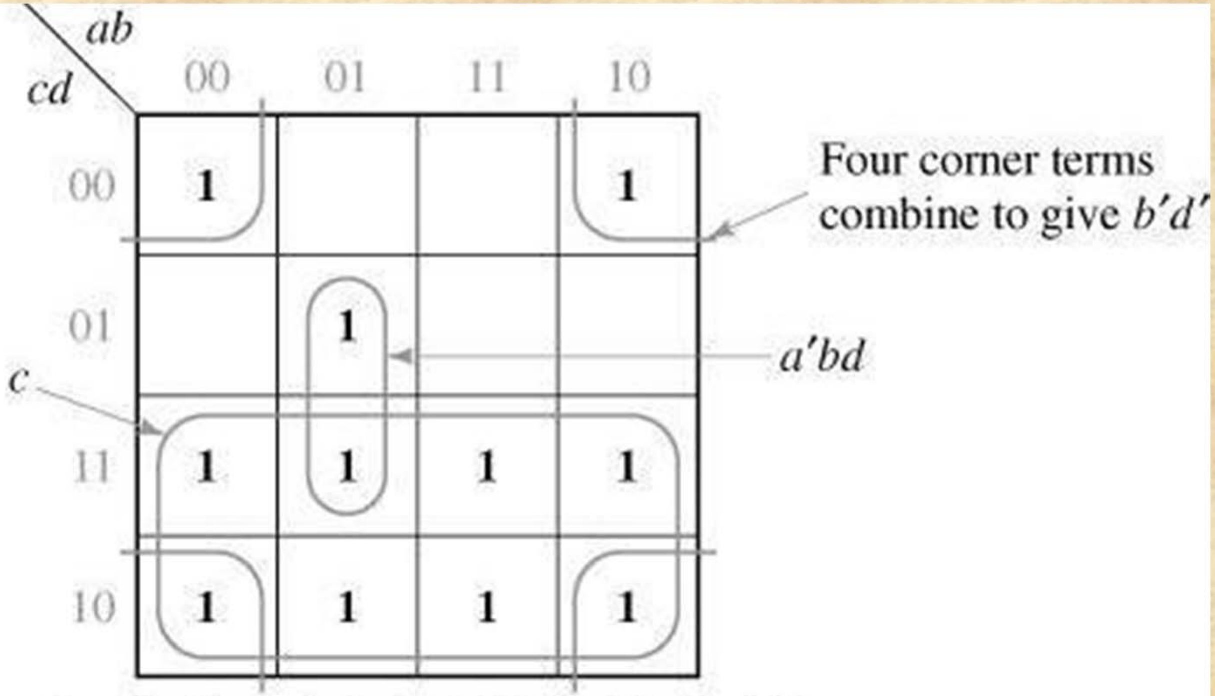
Four-Variable Karnaugh Maps (1/9)

- Are m_8 and m_0 adjacent?
- Are m_2 and m_{10} adjacent?
- Are $loop_{0\&8}$ and $loop_{2\&10}$ adjacent?



Four-Variable Karnaugh Maps (2/9)

- Minterms can be combined in group of 2, 4, or 8 to eliminate 1, 2, or 3 variables.



$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

(b)

Four-Variable Karnaugh Maps (3/9)

	00	01	11	10
00	1	1	1	
01	1	1	1	
11				
10				

Anything wrong?

Four-Variable Karnaugh Maps (4/9)

	0	1
00	1	
01	1	
11	1	
10		

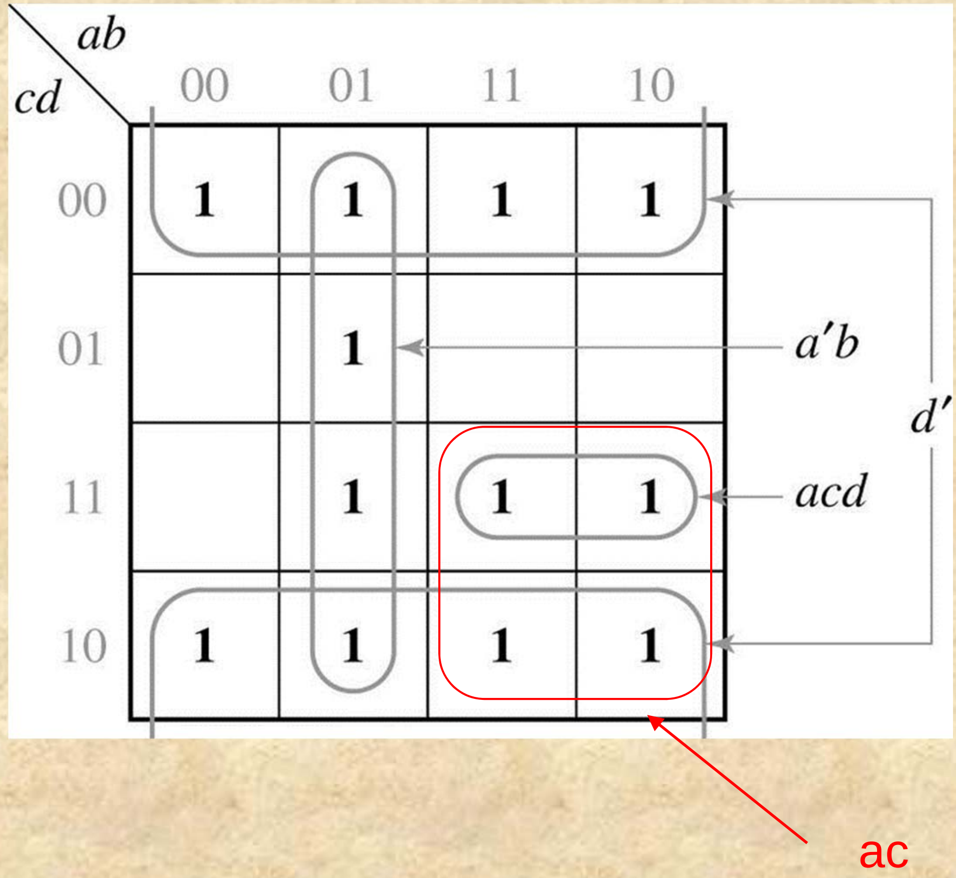
Anything wrong?

Four-Variable Karnaugh Maps (5/9)

- Minterms can be combined in group of 2, 4, or 8 to eliminate 1, 2, or 3 variables.
- The number of minterms, contained in a loop, can only be a power of 2.

Four-Variable Karnaugh Maps (6/9)

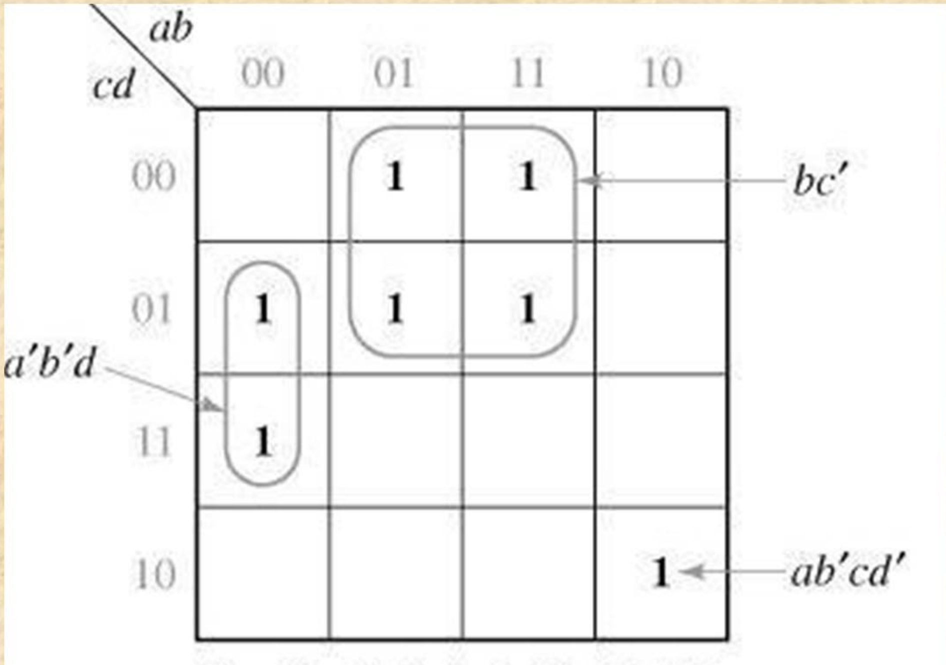
- F(a,b,c,d)=a'b+acd+d'



Can you simplify this further?

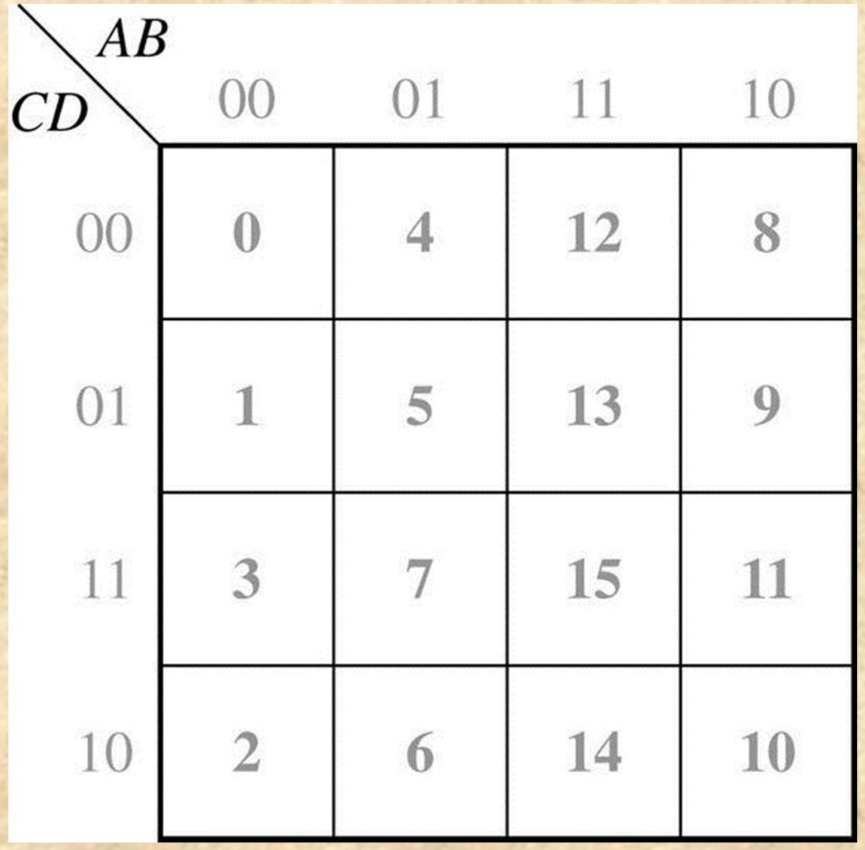
Four-Variable Karnaugh Maps (7/9)

- Minterm Expansion: F(a,b,c,d)=bc’+a’b’d+ab’cd’



$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

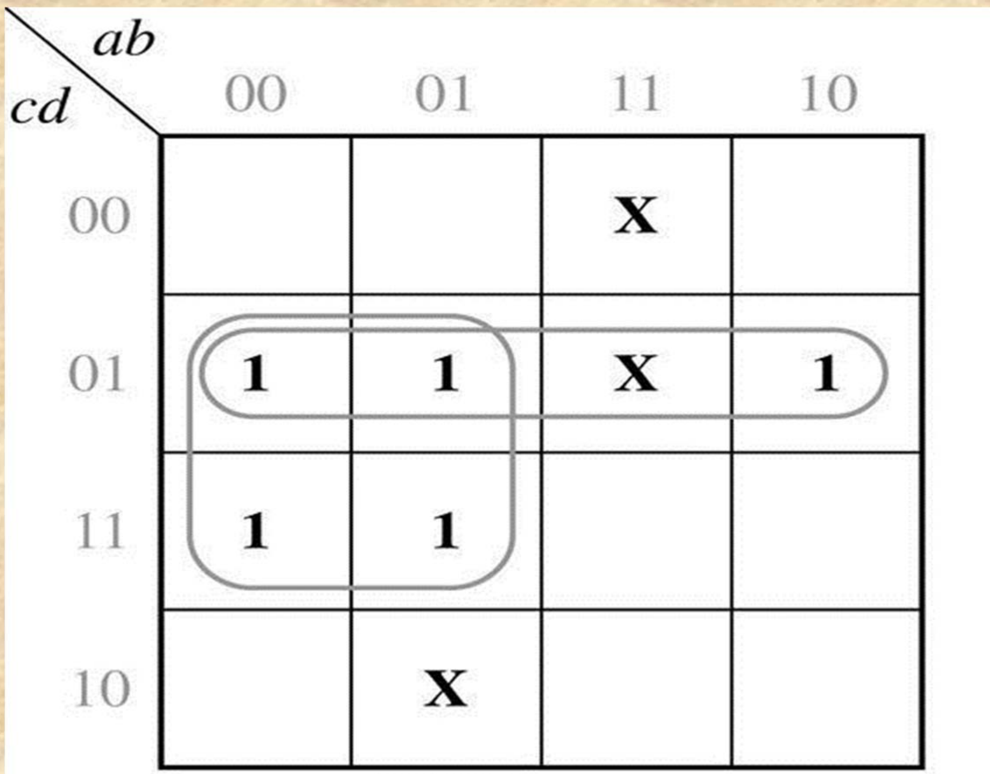
(a)



Four-Variable Karnaugh Maps (8/9)

- Extension to functions with “don’t care” terms
  - “do not care” terms are indicated by X’s in Karnaugh map.
  - The X’s are only used if they will simpify the resulting expression.

Four-Variable Karnaugh Maps (9/9)



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$